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MATHEMATICAL INVESTIGATIONS OF DYNAMICAL SYSTEMS

Final Report
Contract Grant No. ONR N00014-76-C-0073

Submitted to:

Dr. Stuart Brodsky
Director, Mathematics Program
Mathematical and Information Science Division
Office of Naval Research
Department of the Navy
800 North Quincy Street
Arlington, Virginia 22217

Submitted by:

D. L. Lukes
Associate Professor

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901

Report No. UVA/525318/AMCS76/101
September 1976

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For information on current programs and capabilities, write to Director, Research Laboratories for the Engineering Sciences, Thornton Hall, University of Virginia, Charlottesville, Virginia 22901.

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ABSTRACT

This final report summarizes the work accomplished within the Department of Applied Mathematics and Computer Science at the University of Virginia through support of the United States Office of Naval Research under Contract No. N00014-76-C-0073 which expired July 31, 1976. (See Report No. AMCS-3560-102-74, "Mathematical Investigations of Dynamical Systems," for an account of the results obtained under the previous ONR Contract No. N00014-69-A-0060-0010.)

1. Lukes, D. L., "The Geometry of Differential Equations on Banach Space," Calculus of Variations and Control Theory, Academic Press (1976), pp. 93-138.

Summary.

$F(X)$ denotes the class of bounded C^1 vector fields with bounded derivative on an arbitrary Banach space X. An appropriate Banach space $Z(X)$ is selected so as to contain X^F, the collection of solutions X^f to the ordinary differential equations $\dot{x} = f(x)$ on $(-T,T)\times X$ for $f \in F(X)$. This article studies the dependence of X^f upon f and the geometry of X^F in $Z(X)$.

All but the last chapter deal with the problem in which T is finite. The first chapter defines the spaces $F(X)$, $Z(X)$, X^F and studies an appropriate nonlinear operator on $Z(X)$ as a foundation for later sections.

Chapter 2 proves that the map $X^{(\cdot)}:F(X) \to Z(X)$ (which applied mathematicians call "solving the differential equation"), sending $f \to X^f$, is a closed embedding of $F(X)$ into $Z(X)$. In particular $X^{(f)}$ has a continous (Frechet) derivative $X_f^{(f)}$ and X^F is a closed, nonlinear, submanifold of $Z(X)$, modelled upon a Banach space. A corollary shows that $X_f^{(f)}(t,x)$ and the partial derivative $X^{(f)}(t,x)_f$ agree and a variation of parameters formula for computing $X_f^{(f)}$ is thereby attained.

The geometric nature of the embedding is clarified by chapter 3 which recognizes $X^{(\cdot)}$ as a section of a vector bundle $d:Z(X) \to F(X)$. A host of related vector bundle isomorphisms appear.

Chapter 4 describes how the differential equations (i.e. $F(X)$) as well as their solutions lie naturally embedded in $Z(X)$. They are a pair of closed submanifolds of $Z(X)$ whose intersection is given five equivalent geometric characterizations. One of these identifies the intersection with the norm-invariant elements of $F(X)$ under the embedding $X^{(\cdot)}$ and another with the solutions to a quasi-linear partial differential equation in $F(X)$. A general solution to this equation is computed for the cases where X is a finite dimensional or separable Hilbert space. Finally the equation is shown to be the steady state form of a partial differential equation, initial value problem in X which is then solved and shown to satisfy a maximum principle.

The point of view taken in Chapter 5 is that of the calculus of variations. The invariance of curve length under the map $X^{(\cdot)}:F(X) \to Z(X)$ is studied for the situation wherein the vector space $Z(X)$ is turned into a complete Finsler manifold by a change in the measure of curve length which does not disrupt the norm topology of $Z(X)$.

The results of Chapter 6 indicate the existence of some interesting relationships between differential equations on X with those on $F(X)$. This problem brings out the functorial properties of the constructed bundle $d:Z(X) \to F(X)$ and leads to a natural transformation (in the sense of category theory).

Examples in the last chapter show that a loss of differentiability and continuity of $X^{(\cdot)}$ occurs at some points in $F(X)$ when $T = \infty$.

2. Young, D. F., "Delay Equations in Banach Space and the Control of Linear Volterra-Stieltjes Equations," Ph.D. dissertation in the Department of Applied Mathematics and Computer Science, University of Virginia, August 1975, 112 pages.

Summary.

In this dissertation, we study families of linear operators $\{K_\tau : \tau \varepsilon J\}$ and functional equations

(1) $$x = K_\tau x + g$$

where J is any real interval, V is a Banach space and $K_\tau : C(J,V) \to C(J,V)$. The properties of $\{K_\tau\}$ abstract properties of the operator family $\tilde{K}_\tau : C(J,R^n) \to C(J,R^n)$ given by

$$(\tilde{K}_\tau x)(t) = \int_\tau^t [d_s \mu(t,s)]x(s) + \alpha(t,\tau)x(\tau)$$

for each $\tau \varepsilon J$, so that (1) reduces to a Volterra-Stieltjes equation when $V = R^n$. Sufficient conditions on $\{K_\tau\}$ are given which guarantee the existence and uniqueness of a solution to (1). When $V = C([-r,0],R^n)$ and K_τ is defined appropriately, (1) is shown to include as a special case the class of neutral functional equations studied by Hale and Meyer. In this case, our results broaden Hale and Meyer's existence and uniqueness results. In the case $V = R^n$, whenever (1) has a unique solution for all τ and all g, this solution is shown to be of the form

(2) $$x(t) = \Phi_1(t,\tau)g(\tau) + \int_\tau^t \Phi_2(t,s)dg(s).$$

The representation (2) is used to study controllability and time-optimal control of (1), with $g(t) = x_0 + \int_\tau^t (v(s) + B(s)u(s))ds$, where x_0 is the initial state, u is the control function and v(t) and B(t) are given. With compact, but not necessarily convex, control restraints, the relationship between local controllability and global controllability is studied, and a weak form of the bang-bang principle is derived. Under certain conditions (examples illustrate that this does not hold for all cases), it is shown that $\Phi_1(t,s) = \Phi_2(t,s)$ and a sufficient matrix criterion for controllability is derived. With compact convex control restraints, the existence of a time-optimal control function is proven for certain special cases of (1).

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Research is an integral part of the educational program and interests parallel academic specialties. These range from the classical engineering departments of Chemical, Civil, Electrical, and Mechanical to departments of Biomedical Engineering, Engineering Science and Systems, Materials Science, Nuclear Engineering, and Applied Mathematics and Computer Science. In addition to these departments, there are interdepartmental groups in the areas of Automatic Controls and Applied Mechanics. All departments offer the doctorate; the Biomedical and Materials Science Departments grant only graduate degrees.

The School of Engineering and Applied Science is an integral part of the University (approximately 1,400 full-time faculty with a total enrollment of about 14,000 full-time students), which also has professional schools of Architecture, Law, Medicine, Commerce, and Business Administration. In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program. This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service.